Mathematics Model Paper (I-a)

Intermediate I - Year

Time: 3 HRS **Max. Marks: 75**

SECTION - I

I. Answer the following questions. ***10 × 2 = 20***

1. If f = {(4, 5), (5, 6), (6, – 4)} then find i) f + 4 ii) | f |
2. Find the domain of $f(x) = \sqrt{4x - x^2}$
3. Construct a 3×2 matrix whose elements are defined by $a_{ij} = \frac{1}{2}|i - 3j|$
4. A certain book shop has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling prices are Rs. 80, Rs. 60 and Rs. 40 each respectively. Using matrix algebra, find the total value of the books in the shop.
5. If $\bar{a} = 2\hat{i} + 5\hat{j} + \hat{k}$ and $\bar{b} = 4\hat{i} + m\hat{j} + n\hat{k}$ are collinear vectors, then find m, n.
6. OABC is a parallelogram. If $\overrightarrow{OA} = \bar{a}$ and $\overrightarrow{OC} = \bar{c}$, then find the vector equation of the side $\overline{BC}$
7. For what values of λ, the vectors $\hat{i} - \lambda\hat{j} + 2\hat{k}$ and $8\hat{i} + 6\hat{j} - \hat{k}$ are at right angles?
8. If $\tan\theta = \frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ}$ and θ is in Q3 find 'θ'.
9. Find the period of the function f(x) = sin(x + 8x + 27x +... + n^3x).
10. If cosh $x = \frac{5}{2}$, then find

    i) cosh2x ii) sinh2x

SECTION - II

II.Answer any five of the following questions. ***5 × 4 = 20 M***

11. Show that $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$ = abc (a–b) (b-c) (c-a).
12. Let ABCDEF be regular hexagon with centre 'O'. Show that

$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} = 3\overrightarrow{AD} = 6\overrightarrow{AO}$

13. If $|\bar{a}| = |\bar{b}| = 5, (\bar{a}, \bar{b}) = 45^\circ$, then find the area of triangle constructed with the vectors $\bar{a} - 2\bar{b}, 3\bar{a} + 2\bar{b}$ as adjacent sides.

14. If A+B = $\pi/4$, then prove that (1+tanA) (1 + tanB) = 2.

15. Solve $\sqrt{2}$ (sin x + cos x) = $\sqrt{3}$

16. Prove that $\text{Tan}^{-1}\left(\frac{1}{2}\right) + \text{Tan}^{-1}\left(\frac{1}{5}\right) + \text{Tan}^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$

17. In triangle ABC, prove that

$r + r_1 + r_2 - r_3 = 4R\cos C$

SECTION - III

III.Answer any five of the following questions. ***5 × 7 = 35 M***

18. If f : A $\to$ B is a bijective function, then show that $f^{-1}of = I_A$, $fof^{-1} = I_B$.

19. By using mathematical induction show that 1.2.3 + 2.3.4 + 3.4.5 +.... up to n terms =

$$\frac{n(n+1)(n+2)(n+3)}{4} \quad \forall n \in N$$

20. Show that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ac-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = (a^3+b^3+c^3-3abc)^2$

21. Solve the following system of linear equations by using Cramer's rule.

$2x - y + 8z = 13$

$3x + 4y + 5z = 18$

$5x - 2y + 7z = 20$

22. If A = (1, – 2, –1), B = (4, 0, –3) C = (1, 2, –1), D = (2, –4, –5), then find the distance between the lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$

23. If A + B + C = π then prove that $\sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2} = = 1 + 4\sin\left(\frac{\pi-A}{4}\right)\sin\left(\frac{\pi-B}{4}\right)\sin\left(\frac{\pi-C}{4}\right)$

24. Prove that $\frac{r_1}{bc} + \frac{r_2}{ca} + \frac{r_3}{ab} = \frac{1}{r} - \frac{1}{2R}$